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ON GENERALIZED STATISTICAL CONVERGENCE IN TOPOLOGICAL GROUPS

Abstract. In this paper, we introduce the concept of λ-statistical convergence in topological groups. Some inclusion relations between the sets of statistically convergent and λ-statistically convergent sequences are established. Also we introduce the definition of statistically λ-convergence in topological groups and find the relation between these two notions.

1. Introduction

The idea of statistical convergence was formerly given under the name "almost convergence" by Zygmund in the first edition of his celebrated monograph published in Warsaw in 1935 [32]. The concept was formally introduced by Steinhaus [31] and Fast [13] and later was reintroduced by Schoenberg [30], and also independently by Buck [3]. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on it was further investigated from various points of view. For example, statistical convergence has been investigated in summability theory (Boos [2], Çakalli and Khan[5], Çakalli and Thorpe[7], Fridy [15], Šalát [28], Mursaleen and Alotaibi [25], Prullage [27]), topological groups (Çakalli [4], [6]), topological spaces (Di Maio and Kočinac[22]), function spaces (Caserta and Kočinac [8], Caserta, Di Maio and Kočinac [9]), locally convex spaces (Maddox[21]), measure theory (Cheng et al. [10], Connor and Swardson [11], Miller[23]), fuzzy mathematics (Nuray and Savaş [26], Savaş [29]). In the recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions (Antoni [1], Connor and Swardson [11], Connor and Grosse-Erdmann[12]). Mursaleen [24], introduced the λ-statistical convergence for real sequences. For more details on λ-statistical convergence we refer to [16, 17, 18, 19].

The idea is based on the notion of natural density of subsets of $\mathbb{N}$, the set of positive integers, which is defined as follows: The natural density of a subset of $E \subset \mathbb{N}$ is denoted by $\delta(E)$ and is defined by

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} |\{k \le n : k \in E\}|,$$

where the vertical bar denotes the cardinality of the respective set.

DEFINITION 1. *A sequence $x = (x_k)$ of real numbers is said to be statistically convergent to ℓ if for every $\varepsilon > 0$*

$$\delta(\{k \in \mathbb{N} : |x_k - \ell| \ge \varepsilon\}) = 0.$$

In this case, we write $S-\lim x=\ell$ *or* $(x_k)\xrightarrow{S}\ell$ *and* $[S]$ *denotes the set of all statistically convergent sequences.*

Let $\lambda=(\lambda_m)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{m+1}\le\lambda_m+1,\lambda_1=1.$$

The generalized de la Vallée-Poussin mean of a sequence $x=(x_k)$ is defined by

$$t_m(x)=\frac{1}{\lambda_m}\sum_{k\in I_m}x_k,$$

where $I_m=[m-\lambda_m+1,m]$.
This notation for λ and I_m we keep throughout the paper.

DEFINITION 2. *([20]) A sequence* $x=(x_k)$ *is said to be* (V,λ)*-summable to a number* ℓ *if*

$$t_m(x)\to\ell,\ \ as\ \ m\to\infty.$$

In this case we write $\ell=\lambda-\lim x$. *If* $\lambda_m=m$, *then* (V,λ)*-summability reduces to* $(C,1)$*-summability. We write*

$$[C,\lambda]=\left\{x=(x_k):\exists\,\ell\in\mathbb{R},\lim_{m\to\infty}\frac{1}{m}\sum_{k=1}^{m}|x_k-\ell|=0\right\}$$

and

$$[V,\lambda]=\left\{x=(x_k):\exists\,\ell\in\mathbb{R},\lim_{m\to\infty}\frac{1}{\lambda_m}\sum_{k\in I_m}|x_k-\ell|=0\right\}$$

for the sets of sequences $x=(x_k)$ *which are strongly Cesàro summable (see [14]) and strongly* (V,λ)*-summable to* ℓ, *i.e.* $(x_k)\xrightarrow{[C,1]}\ell$ *and* $(x_k)\xrightarrow{[V,\lambda]}\ell$, *respectively.*

Let $K\subseteq\mathbb{N}$ be a set of positive integers. Then

$$\delta_\lambda(K)=\lim_m\frac{1}{\lambda_m}|\{m-\lambda_m+1\le k\le m:k\in K\}|$$

is said to be λ-*density* of K provided the limit exists.

DEFINITION 3. *([24]) A sequence* $x=(x_k)$ *is said to be* λ*-satistically convergent or* S_λ*-convergent to* ℓ *if for every* $\varepsilon>0$

$$\lim_{m\to\infty}\frac{1}{\lambda_m}|\{k\in I_m:|x_k-\ell|\ge\varepsilon\}|=0,$$

or equivalently a sequence $x=(x_k)$ *is said to be* λ*-satistically convergent to* ℓ *or* S_λ*-convergent to* ℓ *if for every* $\varepsilon>0$ *the set* $K_\varepsilon=\{k\in\mathbb{N}:|x_k-\ell|\ge\varepsilon\}$ *has* λ*-density* 0. *In this case we write* $S_\lambda-\lim x=\ell$ *or* $(x_k)\xrightarrow{S_\lambda}\ell$ *and*

$$[S_\lambda]=\{x=(x_k):\exists\,\ell\in\mathbb{R},S_\lambda-\lim x=\ell\}.$$

It is clear that if $\lambda_m = m$, then $[S_\lambda]$ is same as $[S]$.

The purpose of this article is to give certain characterizations of λ-statistically convergent sequences in topological groups and to obtain extensions of a decomposition theorem and some inclusion results related to the notions statistically convergent and λ-statistically convergent sequences in topological groups.

2. λ-statistical convergence in topological groups

Throughout the article X will denote a topological Hausdorff group, written additively, which satisfies the first axiom of countability. The identity element of X will be denoted by $\mathbf{0}$. Now we give the definitions of λ-statistically convergence in topological groups.

DEFINITION 4. *A sequence (x_k) of points in X is said to be λ-statistically convergent or S_λ-convergent to an element x_0 of X if for each neighbourhood V of $\mathbf{0}$,*

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V\}) = 0$$

i.e.

$$\lim_m \frac{1}{\lambda_m} |\{k \in I_m : x_k - x_0 \notin V\}| = 0.$$

In this case, we write $S_\lambda - \lim_{k\to\infty} x_k = x_0$ or $(x_k) \xrightarrow{S_\lambda} x_0$ and we define

$$[S_\lambda(X)] = \{(x_k) : \text{ for some } x_0, \ S_\lambda - \lim_{k\to\infty} x_k = x_0\}.$$

In particular,

$$\left[S_\lambda^0(X)\right] = \{(x_k) : \ S_\lambda - \lim_{k\to\infty} x_k = 0\}.$$

DEFINITION 5. *A sequence (x_k) of points in X is said to be λ-statistically Cauchy or S_λ-Cauchy in X if for each neighbourhood V of $\mathbf{0}$, there is an integer $n(V)$ such that*

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_{n(V)} \notin V\}) = 0.$$

Throughout the article $[s(X)]$, $[S_\lambda(X)]$ and $[C_\lambda(X)]$ denote the set of all, S_λ-convergent and S_λ-Cauchy sequences in X, respectively.

THEOREM 1. *Every λ-statistically convergent sequence in X has only one limit.*

Proof. Suppose that a sequence $x = (x_k)$ in X has two different λ-statistical limits, x_0 and y_0, say. Since X is a Hausdorff space there exists a neighbourhood V of 0 such that $x_0 - y_0 \notin V$. Also for each neighbourhood V of $\mathbf{0}$ there exists a symmetric neighbourhood W of $\mathbf{0}$ such that $W + W \subset V$. Write $z_k = x_0 - y_0$ for all $k \in \mathbb{N}$. Therefore for all $m \in \mathbb{N}$,

$$\{k \in I_m : z_k \notin V\} \subset \{k \in I_m : x_0 - x_k \notin W\} \cup \{k \in I_m : x_k - y_0 \notin W\}.$$

Now it follows from this inclusion that, for all $m \in \mathbb{N}$,

$$\frac{1}{\lambda_m}|\{k \in I_m : z_k \notin V\}| \leq \frac{1}{\lambda_m}|\{k \in I_m : x_0 - x_k \notin W\}| + \frac{1}{\lambda_m}|\{k \in I_m : x_k - y_0 \notin W\}|.$$

Since $S_\lambda - \lim x_k = x_0$ and $S_\lambda - \lim x_k = y_0$, we get

$$\lim_{m\to\infty}\frac{1}{\lambda_m}|\{k \in I_m : z_k \notin V\}| \leq \lim_{m\to\infty}\frac{1}{\lambda_m}|\{k \in I_m : x_0 - x_k \notin W\}| + \lim_{m\to\infty}\frac{1}{\lambda_m}|\{k \in I_m : x_k - y_0 \notin W\}|.$$

Hence $1 \leq 0+0 = 0$. This contradiction shows that $x_0 = y_0$. □

THEOREM 2. *A sequence (x_k) is S_λ-convergent to x_0 in X if and only if for each neighbourhood V of $\mathbf{0}$ there exists a subsequence $(x_{k'(m)})$ of (x_k) such that $\lim_{m\to\infty} x_{k'(m)} = x_0$ and*

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_{k'(m)} \notin V\}) = 0.$$

Proof. Let $x = (x_k)$ be a sequence in X such that $S_\lambda - \lim_{k\to\infty} x_k = x_0$. Let $\{V_n\}$ be a nested base of neighbourhoods of $\mathbf{0}$. We write $E^{(i)} = \{k \in \mathbb{N} : x_k - x_0 \notin V_i\}$ for any positive integer i. Then for each i, we have $E^{(i+1)} \subset E^{(i)}$ and $\lim \frac{|E^{(i)} \cap I_m|}{\lambda_m} = 1$. Choose $n(1)$ such that $m > n(1)$, then $|E^{(1)} \cap I_m| > 0$ i.e., $E^{(1)} \cap I_m \neq \phi$. Then for each positive integer m such that $n(1) \leq m < n(2)$, choose $k'(m) \in I_m$ such that $k'^{(i)} \in I_m$ i.e. $x_{k'(m)} - x_0 \in V_1$. In general, choose $n(p+1) > n(p)$ such that $m > n(p+1)$, then $E^{(p+1)} \cap I_m \neq \phi$. Then for all m satisfying $n(p) \leq m < n(p+1)$, choose $k'^{(p)} \in I_m$ i.e. $x_{k'(m)} - x_0 \in V_p$. Hence it follows that $\lim_m x_{k'(m)} = x_0$. Let V be any neighbourhood of $\mathbf{0}$. Then there is a symmetric neighbourhood W of $\mathbf{0}$ such that $W + W \subset V$. Now we have

$$\{k \in \mathbb{N} : x_k - x_{k'(m)} \notin V\} \subseteq \{k \in \mathbb{N} : x_k - x_0 \notin W\} \cup \{m \in \mathbb{N} : x_{k'(m)} - x_0 \notin W\}.$$

Since $S_\lambda - \lim_{k\to\infty} x_k = x_0$ and $\lim_{m\to\infty} x_{k'(m)} = x_0$, this implies that

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_{k'(m)} \notin V\}) = 0.$$

Conversely, suppose for each neighbourhood V of $\mathbf{0}$ there exists a subsequence $(x_{k'(m)})$ of (x_k) such that $\lim_{m\to\infty} x_{k'(m)} = x_0$ and

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_{k'(m)} \notin V\}) = 0.$$

Since V is a neighbourhood of $\mathbf{0}$, we may choose a symmetric neighbourhood W of $\mathbf{0}$ such that $W + W \subset V$. Then we have

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V\}) \leq \delta_\lambda(\{k \in \mathbb{N} : x_k - x_{k'(m)} \notin W\}) + \delta_\lambda(\{k \in \mathbb{N} : x_{k'(m)} - x_0 \notin W\}).$$

Therefore

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V\}) = 0.$$

This completes the proof of the theorem. □

THEOREM 3. *If* $\lim_{k\to\infty} x_k = x_0$ *and* $S_\lambda - \lim_{k\to\infty} y_k = 0$, *then* $S_\lambda - \lim_{k\to\infty}(x_k + y_k) = x_0$.

Proof. Let V be any neighbourhood of $\mathbf{0}$. Then we may choose a symmetric neighbourhood W of $\mathbf{0}$ such that $W + W \subset V$. Since $\lim_{k\to\infty} x_k = x_0$, then there exists an integer n_0 such that $k \geq n_0$ implies that $x_k - x_0 \in W$. Hence

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin W\}) = 0.$$

By assumption $S_\lambda - \lim_{k\to\infty} y_k = 0$, then we have $\delta_\lambda(\{k \in \mathbb{N} : y_k \notin W\}) = 0$. Thus

$$\delta_\lambda(\{k \in \mathbb{N} : (x_k - x_0) + y_k \notin V\}) \leq \delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin W\}) + \delta_\lambda(\{k \in \mathbb{N} : y_k \notin W\}).$$

This implies that $S_\lambda - \lim_{k\to\infty}(x_k + y_k) = \lim_{k\to\infty} x_k$. □

THEOREM 4. *If a sequence* (x_k) *is* S_λ*-convergent to* x_0 *in* X, *then there are sequences* (y_k) *and* (z_k) *such that* $x_k = y_k + z_k$, *for each* $k \in \mathbb{N}$,$S_\lambda - \lim_{k\to\infty} y_k = x_0$, $\delta_\lambda(\{k \in \mathbb{N} : x_k \neq y_k\}) = 0$ *and* (z_k) *is a* S_λ*-null sequence*.

Proof. Let $\{V_i\}$ be a nested base of neighbourhood of $\mathbf{0}$. Take $n_0 = 0$ and choose an increasing sequence (n_i) of positive integers such that

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V_i\}) < \frac{1}{i} \text{ for } k > n_i.$$

Let us define the sequences (y_k) and (z_k) as follows:

$$y_k = x_k \quad \text{and} \quad z_k = 0, \quad \text{if} \quad 0 < k \leq n_1;$$

$$y_k = x_k \quad \text{and} \quad z_k = 0, \quad \text{if} \quad x_k - x_0 \in V_i$$

$$y_k = x_0 \quad \text{and} \quad z_k = x_k - x_0, \quad \text{if} \quad x_k - x_0 \notin V_i.$$

We have to show that (i) $\lim_{k\to\infty} y_k = x_0$ (ii) (z_k) is an S_λ-null sequence.
(i) Let V be any neighbourhood of $\mathbf{0}$. We may choose a positive integer i such that $V_i \subset V$. Then $y_k - x_0 = x_k - x_0 \in V_i$, for $k > n_i$.If $x_k - x_0 \notin V_i$, then $y_k - x_0 = x_0 - x_0 = 0 \in V$. Hence $\lim_{k\to\infty} y_k = x_0$.
(ii) It is enough to show that $\delta_\lambda(\{k \in \mathbb{N} : z_k \neq 0\}) = 0$. For any neighbourhood V of $\mathbf{0}$, we have

$$\delta_\lambda(\{k \in \mathbb{N} : z_k \notin V\}) \leq \delta_\lambda(\{k \in \mathbb{N} : z_k \neq 0\}).$$

If $n_p < k \leq n_{p+1}$, then

$$\{k \in \mathbb{N} : z_k \neq 0\} \subset \{k \in \mathbb{N} : x_k - x_0 \notin V_p\}.$$

If $p > i$ and $n_p < k \leq n_{p+1}$, then

$$\delta_\lambda(\{k \in \mathbb{N} : z_k \neq 0\}) \leq \delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V_p\}) < \frac{1}{p} < \frac{1}{i} < \varepsilon.$$

This implies that $\delta_\lambda(\{k \in \mathbb{N} : z_k \neq 0\}) = 0$. Hence (z_k) is an S_λ-null sequence. □

THEOREM 5. *Let $x = (x_k)$ be a sequence in X. If there is a S_λ-convergent sequence $y = (y_k)$ in X such that $\delta_\lambda(\{k \in \mathbb{N} : y_k \neq x_k \notin V\}) = 0$ then x is also S_λ-convergent.*

Proof. Suppose that $\delta_\lambda(\{k \in \mathbb{N} : y_k \neq x_k \notin V\}) = 0$ and $S_\lambda - \lim y_k = x_0$. Then for every neighbuohood V of $\mathbf{0}$, we have

$$\delta_\lambda(\{k \in \mathbb{N} : y_k - x_0 \notin V\}) = 0.$$

Now,

$$\{k \in \mathbb{N} : x_k - x_0 \notin V\} \subseteq \{k \in \mathbb{N} : y_k \neq x_k \notin V\} \cup \{k \in \mathbb{N} : y_k - x_0 \notin V\}$$

$$\Rightarrow \delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V\}) \leq \delta_\lambda(\{k \in \mathbb{N} : y_k \neq x_k \notin V\}) + \delta_\lambda(\{k \in \mathbb{N} : y_k - x_0 \notin V\}).$$

Therefore we have

$$\delta_\lambda(\{k \in \mathbb{N} : x_k - x_0 \notin V\}) = 0.$$

This completes the proof of the theorem. □

THEOREM 6. *Let $x = (x_k)$ be sequence in X. Then $(x_k) \xrightarrow{S} x_0$ implies $(x_k) \xrightarrow{S_\lambda} x_0$ if $\liminf_m \frac{\lambda_m}{m} > 0$.*

Proof. Suppose first that $\liminf_m \frac{\lambda_m}{m} > 0$ and $(x_k) \xrightarrow{S} x_0$. Let V be any neighbourhood of $\mathbf{0}$. Then for all $m \in \mathbb{N}$,

$$\frac{1}{m}|\{k \leq m : x_k - x_0 \notin V\}| \geq \frac{1}{m}|\{k \in I_m : x_k - x_0 \notin V\}|$$

$$\geq \frac{\lambda_m}{m}\frac{1}{\lambda_m}|\{k \in I_m : x_k - x_0 \notin V\}|$$

since $(x_k) \xrightarrow{S} x_0$. Therefore this inequality implies that $(x_k) \xrightarrow{S_\lambda} x_0$, i.e. $[S] \subset [S_\lambda]$. □

THEOREM 7. *Let $x = (x_k)$ be sequence in X. Then $S = S_\lambda$ if $\lim_m \frac{\lambda_m}{m} = 1$.*

Proof. Suppose that $\lim_m \frac{\lambda_m}{m} = 1$. Let V be any neighbourhood of $\mathbf{0}$. We observe that

$$\frac{1}{m}|\{k \leq m : x_k - x_0 \notin V\}| \leq \frac{1}{m}|\{k \leq m - \lambda_m : x_k - x_0 \notin V\}| + \frac{1}{m}|\{k \in I_m : x_k - x_0 \notin V\}|$$

$$\leq \frac{m - \lambda_m}{m} + \frac{1}{m}|\{k \in I_m : x_k - x_0 \notin V\}|.$$

$$= \frac{m - \lambda_m}{m} + \frac{\lambda_m}{m}\frac{1}{\lambda_m}|\{k \in I_m : x_k - x_0 \notin V\}|.$$

This imples that x is statistically convergent to x_0 in X if x is λ-statistically convergent to x_0. Thus $[S_\lambda] \subset [S]$.
Since $\lim_m \frac{\lambda_m}{m} = 1$ implies that $\liminf_m \frac{\lambda_m}{m} > 0$, then from Theorem 5, we have $S \subset S_\lambda$. Hence $[S] = [S_\lambda]$.

□

3. statistical λ-convergence in topological groups

DEFINITION 6. *A sequence (x_k) of points in X is said to be statistically λ-convergent or S_{δ_λ}-convergent to an element x_0 of X if for each neighbourhood V of $\mathbf{0}$, the set $K(\lambda) = \{n \in \mathbb{N} : t_n(x) - x_0 \notin V\}$ has natural density zero, or equivalently, $\delta(K(\lambda)) = 0$, i.e.*

$$\lim_m \frac{1}{m} |\{n \le m : t_n(x) - x_0 \notin V\}| = 0.$$

In this case, we write $S_{\delta_\lambda} - \lim_{k\to\infty} x_k = x_0$ or $(x_k) \xrightarrow{S_{\delta_\lambda}} x_0$ and we define

$$\left[S_{\delta_\lambda}(X)\right] = \{(x_k) : \text{ for some } x_0,\ S_{\delta_\lambda} - \lim_{k\to\infty} x_k = x_0\}.$$

THEOREM 8. *A sequence $x = (x_k)$ in X is statistically λ-convergent to x_0 if and only if there exists a set $K = \{k_1 < k_2 < \cdots < k_m < \ldots\} \subseteq \mathbb{N}$ such that $\delta(K) = 1$ and $\lambda - \lim x_{k_m} = x_0$.*

Proof. Suppose that there exists a set $K = \{k_1 < k_2 < \cdots < k_m < \ldots\} \subseteq \mathbb{N}$ such that $\delta(K) = 1$ and $\lambda - \lim x_{k_m} = x_0$. For each neighbourhood V of $\mathbf{0}$, there exists a positive integer N such that $(t_{n_m}(x) - x_0) \in V$ for $m > N$.

We put $K(\lambda) = \{m \in \mathbb{N} : t_m(x) - x_0 \notin V\}$ and $K_1 = \{k_{N+1}, k_{N+2}, \ldots\}$. Then $\delta(K_1) = 1$ and $K(\lambda) \subseteq \mathbb{N} - K_1$ which implies that $\delta(K(\lambda)) = 0$. Hence $x = (x_k)$ is statistically λ-convergent to x_0.
Conversely, suppose that $x = (x_k)$ is statistically λ-convergent to x_0. Let $\{V_r\}$ be a nested base of neighbourhoods of $\mathbf{0}$. We put

$$K_r(\lambda) = \{m \in \mathbb{N} : t_{k_m}(x) - x_0 \notin V_r\} \text{ and } M_r(\lambda) = \{m \in \mathbb{N} : t_{k_m}(x) - x_0 \in V_r\}.$$

Then $\delta(K_r(\lambda)) = 0$,

$$M_1(\lambda) \supset M_2(\lambda) \supset \cdots \supset M_r(\lambda) \supset M_{r+1}(\lambda) \supset \ldots \tag{1}$$

and

$$\delta(M_r(\lambda)) = 1, r \in \mathbb{N}. \tag{2}$$

We show that for $m \in M_r(\lambda)$, $\lambda - \lim x_{k_m} = x_0$. Suppose that (x_{k_m}) is not λ-convergent to x_0. For each neighbourhood V of $\mathbf{0}$, there exists a positive integer N such that $(t_{n_m}(x) - x_0) \notin V$ for $m > N$.
Let $M(\lambda) = \{m \in \mathbb{N} : t_{k_m}(x) - x_0 \notin V\}$ and $V \supset V_r$ for $r \in \mathbb{N}$. Then $\delta(M(\lambda)) = 0$. By (1), we have $M_r(\lambda) \subset M(\lambda)$ and hence $\delta(M_r(\lambda)) = 0$, which contradicts (2). Therefore (x_{k_m}) is λ-convergent to x_0. This completes the proof of the theorem. □

THEOREM 9. *A sequence $x = (x_k)$ in X is λ-statistically convergent to x_0 if and only if there exists a set $K = \{k_1 < k_2 < \cdots < k_m < \ldots\} \subseteq \mathbb{N}$ such that $\delta_\lambda(K) = 1$ and $\lim x_{k_m} = x_0$.*

Proof. Proof of the theorem follows from Theorem 2 and Theorem 8. □

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